VII. The normal approximation to a binomial distribution

A. In chapter 9, a Poisson distribution was used to approximate a binomial distribution when $n \geq 30$ and either $np < 5$ or $nq < 5$.

B. Here we learn how the standard normal distribution may be used to approximate a binomial distribution when $n \geq 30$ and both np and nq are ≥ 5.

C. Linda wants to know how many sales will result from calls to 40 previous customers. Past history indicates 25% of these customers will make an additional purchase. Calculate the probability of making at least 12 sales.

1. Using the binomial would require solving $P_{\text{binomial}}(X) = \frac{n!}{X!(n-x)!}p^x q^{n-x}$ many times.

2. The normal approximation to the binomial distribution is appropriate to solve this problem.
   a. $n \geq 30$ as $n = 40$
   b. np and nq are ≥ 5 as $np = 40 \times .25 = 10$ and $nq = 40 \times .75 = 30$

3. The mean and standard deviation would be calculated as follows:

$\mu = np = (40)(.25) = 10$

$\sigma = \sqrt{npq} = \sqrt{(40)(.25)(.75)} = 2.7386$

Note: Mean and standard deviation formulas for a binomial distribution were given on page 56.

4. The **continuity correction factor**
   a. Because a discrete event (12 sales) has to be considered a continuous interval when using the continuous normal probability distribution, the number 12 must be expressed as the interval of 11.5 to 12.5. This is done to ensure that the entire area under a normal curve is included in the analysis.
   b. Linda's question includes 12 so the lower limit of 11.5 is appropriate. Had she excluded 12 with $p(x) > 12$, then 12.5 would have been the value of x.

5. Calculate $P(x \geq 12)$ using the normal approximate of the binomial.

$$Z = \frac{x-\mu}{\sigma} = \frac{11.5-10}{2.7386} = \frac{1.5}{2.7386} = .55 \rightarrow .2088 \text{ and } 50\% - 20.88\% = 29.12\%$$

VIII. Summary of problem types

A. Finding the probability given a range for the random variable

The problems on page 59 describe situations where the **range is known** and the probability for that range must be determined.

1. Calculate z using this formula. $Z = \frac{x-\mu}{\sigma}$
2. Find z in the margins of the z table.
3. Look in the body of the table to find the probability.
4. Continue until the problem is solved.

B. Finding a range for the random variable given a probability

The problems on page 60 describe situations where the **probability is known**, and the range for that probability must be determined.

1. Find the probability in the body of the z table.
2. Look to the margins of the table to find z.
3. Find the range for x using this formula. $\mu \pm z\sigma$
4. Continue until the problem is solved.