Chapter 21 Nonparametric Hypothesis Testing of Ordinal Data Part I

I. A run test is used to determine randomness based upon order of occurrence.

A. To be successful, an experiment often requires data be randomly collected.
   1. Inferential statistics often requires data be collected randomly.
   2. Quality control, studied in chapter 17, requires defect testing be done to randomly selected items.

B. Data studied pertains to a two category variable (male/female, pass/fail, etc.). The number of runs (similar observations) determines randomness. Too many or too few runs causes rejection of the null hypothesis.

C. Linda wants an .05 level test to determine whether the gender of people walking into her store is a random event.
   1. This gender data was collected from Linda's Saturday morning customers. Runs have been underlined.
   2. F F F, M M, F F F F, M, F F F F F, M M M M, F, M M M M, F F F F F, M M M M M, F F, M M, F F F

The sample size of either category is n_1.

The sample size of the other category is n_2.

The number of runs is r. The sampling distribution of r is approximately normal provided the sample size of either category (n_1 or n_2) is beyond 20. If both are ≤ 20, tables containing the critical value of r should be used.

Here are the mean and standard error associated with the sampling distribution of r.

$$\mu_r = \frac{2n_1n_2}{n_1+n_2}+1 \qquad \sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n+n_2-1)}}$$

$$z = \frac{r-\mu_r}{\sigma_r}$$

The test statistic is r. If z from the test statistic is beyond the critical value of z, the null hypothesis is rejected.

$n_1 = 23$ females	$n_2 = 18$ males	$r = 13$ runs

$$\mu_r = \frac{2n_1n_2}{n_1+n_2}+1 = \frac{2(23)(18)}{23+18}+1 = \frac{828}{41}+1 = 21.195$$

$$\sigma_r = \sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n_1+n_2-1)}} = \sqrt{\frac{2(23)(18)[(2(23)(18)-23-18)]}{(23+18)^2(23+18-1)}} = \sqrt{\frac{651{,}636}{67{,}240}} = 3.113$$

$$Z = \frac{r-\mu_r}{\sigma_r} = \frac{13.000-21.195}{3.113} = -2.63$$

For the .05 level of significance, z is ±1.96 for this two-tail test. Reject H_0 because -2.63 is beyond - 1.96. Gender of customers walking into Linda's store is not random.

D. Run tests may be done using the median. Runs consist of consecutive outcomes larger or smaller than the median. Outcomes equal to the median are ignored.

II. One-tail testing of one sample median using the sign test

A. This test is equivalent to a one-tail parametric test of 1 sample mean.

B. Data must be at least ordinal in nature and knowledge about the shape of the distribution is not required.

C. A (+) sign is assigned to values above the median of interest and a (-) sign to those below the median. Those equal to the median are dropped from the test and n is reduced accordingly.

D. Our study of inferential statistics began when Linda became concerned about a drop in the average customer purchase from \$7.75. If Linda does not know the shape of the distribution, she can do a sign test of this year's data against last year's median of \$7.70. Median hourly sale for 7 randomly selected periods will be tested at the .05 level of significance.
   1. If the median has decreased, the proportion of (-) signs should be greater than the proportion of (+) signs.
   2. H_o: $p \geq .50$ and H_1 : $p < .50$ (H_1 must be less-than because this is the change being tested.)
      a. For small samples, the binomial distribution is used to calculate the probability of the distribution tail (observations beyond the proposed median).
      b. P (often called π) equals .5, n equals total observations, and x equals observations beyond the proposed median. If the probability of the tail is less than the level of significance (alpha), the null hypothesis is rejected. With a two-tail test, the probability calculation is doubled.
   3. Z is appropriate for large samples with p equal to .50 (see section IC of page 94).
   4. The p-value approach to hypothesis testing will be used with these sign tests.
      a. Five median sales figures are below \$7.70 and n is 6 because of a tie.
      b. The binomial table (ST 1) yields the following: $P(x \geq 5) = .094 + .016 = .11$.
      c. Accept H_o as .11 is greater than .05. Chance could have caused these decreases.
      d. With samples of 6, all must decrease to reject H_o. $P(x = 6) = .016$ and $.016 < .05$)

Sample	Median	Sign
1	\$7.65	-
2	\$7.50	-
3	\$8.00	+
4	\$7.60	-
5	\$7.70	0
6	\$7.35	-
7	\$7.55	-