IV. Two-tail testing of two sample proportions

A. Many interesting problems involve two population proportions.
   1. Does consumer satisfaction differ because of gender, age, income, etc.?
   2. Does machine A produce fewer defects than machine B?
   3. Does taking a certain drug lower the incidence of illness?

B. A two-tail problem
   1. Linda wants to know at .05 level of significance whether two of her stores have equal levels of customer satisfaction. Store #1 had 80 of 100 satisfied customers while store #2 had 45 of 50 satisfied customers.
   2. The 5-step approach to hypothesis testing
      a. The null hypothesis and alternate hypothesis are:
         1) $H_0 : p_1 = p_2$
         2) $H_1 : p_1 \neq p_2$
      b. The level of significance will be .05 and $\alpha/2 = .05/2 = .025 \rightarrow z = \pm 1.96$.
      c. The test statistic will be $\bar{p}$.

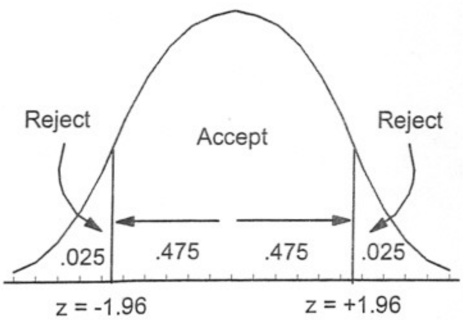
n_1 is sample size #1 and x_1 is successful responses from this sample.
n_2 is sample size #2 and x_2 is successful responses from this sample.
$\bar{p}_1$, the sample proportion for population # 1, is $\frac{x_1}{n_1} = \frac{80}{100} = .80$.
$\bar{p}_2$, the sample proportion for population # 2, is $\frac{x_2}{n_2} = \frac{45}{50} = .90$.
$\bar{p}_w$ is the weighted or pooled estimate of the population mean.

$$\bar{p}_w = \frac{\text{total successes}}{\text{total sampled}} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\frac{\bar{p}w(1-\bar{p}w)}{n_1} + \frac{\bar{p}w(1-\bar{p}w)}{n_2}}}$$

      d. The decision rule will be, if z from the test statistic is beyond the critical value of z, the null hypothesis will be rejected.
      e. Apply the decision rule.

$$\bar{p}_w = \frac{x_1 + x_2}{n_1 + n_2} = \frac{80 + 45}{100 + 50} = .833$$



$$z = \frac{\bar{p}_1 - \bar{p}_2}{\sqrt{\frac{\bar{p}w(1-\bar{p}w)}{n_1} + \frac{\bar{p}w(1-\bar{p}w)}{n_2}}}$$

$$= \frac{.80 - .90}{\sqrt{\frac{.833(1-.833)}{100} + \frac{.833(1-.833)}{50}}}$$

$$= -1.55$$

Accept H_0 because -1.55 is not beyond -1.96. Customer satisfaction is the same at the .05 level of significance.

The p-value method yields the same answer.
$z = -1.55 \rightarrow .4394$ and $.5000 - .4394 = .0606$ for one tail
Accept H_0 because $P = 2(.0606) = .1212$ and $.1212 > .05$.

V. One-tail testing of two sample proportions

A. One-tail problems involve change in one direction.

B. Doing the above problem as a one-tail problem, the question could be; does store #2 give better service?

$H_0 : p_2 \leq p_1$ and $H_1 : p_2 > p_1$

   1. Using z yields the following analysis.

Accept H_0 because $\alpha = .05 \rightarrow z$ of ± 1.645 and -1.55 is not beyond -1.645.

   2. The p method yields the following analysis.

$z = -1.55 \rightarrow .4394$ and $p = .5000 - .4394 = .0606$
Accept H_0 because $.0606 > .05$.