

PART V

THE PRESENT AND FUTURE VALUE OF MONEY

Interest is the price paid for the use of money over a period of time. It is logical to think of interest as an accumulation added to an initial value, the principal, with the resulting sum or amount growing larger with time. Compound interest results when interest is paid on interest resulting in much faster growth than with simple interest, where compounding does not take place. With an annuity, growth is even faster as a number of equal payments (principals) are added every equal time period to the compounding process.

Present Value is the interest accumulation process in reverse. Rather than adding interest to a principal to determine a sum, it is in effect subtracted from a sum to determine a principal. Your accumulation loses value as you move from some point in the future back towards the present. Value at the beginning of a time line is the Present Value and value at the end of a time line is the Future Value, often called the Sum. These concepts will become more understandable as you study the following practical problems.

LET P = Present Value or Principal
 F = Future Value or Sum
 n = number of time periods
 i = interest rate per period
 I = Interest Earned
 A = Annuity's Equal Payments
 > = more than

PVM = Present Value Multiple
 FVM = Future Value Multiple
 PVMA = Present Value Multiple Annuity
 FVMA = Future Value Multiple Annuity

Note: These are labels which will be looked up in interest tables on the next two pages.

INTEREST FORMULAS AND SAMPLE PROBLEMS

Note: Students will find it easier to study the Future Value Analysis on the right before the Present Value Analysis on the left. Problems B. and C. require use of the Tables on the next page.

- A. **Simple Interest** (one payment, one interest calculation) Problem: Calculate the Present Value of \$116 to be received in one year and the Future Value in one year of \$100 today. Use 16% simple interest.

Given: F = \$116
 i = 16%
 n = 1 year
 P = _____



$$\begin{aligned}
 P &= F - I \\
 &= F - (Pin) \\
 &= \$116 - (\$100)(.16)(1) \\
 &= \$116 - \$16 \\
 &= \$100
 \end{aligned}$$

Given: P = \$100
 i = 16%
 n = 1 year
 F = _____

$$\begin{aligned}
 F &= P + I \\
 &= P + (Pin) \\
 &= \$100 + (\$100)(.16)(1) \\
 &= \$100 + \$16 \\
 &= \$116
 \end{aligned}$$

Note: \$116 future dollars are worth \$100 in the present, and \$100 of present dollars are \$116 future dollars.

- B. **Compound Interest** (one payment, > 1 interest calculation) Problem: Calculate the Present Value of \$117 to be received in one year and the Future Value in one year of \$100 today. Use 16% interest compounded quarterly.

Given: F = \$117
 i = 16% / 4 = 4%
 n = (1)(4) = 4 qtrs.
 P = _____



$$\begin{aligned}
 P &= F(PVM) \\
 &= 117(.8548) \text{ see table} \\
 &= 100
 \end{aligned}$$

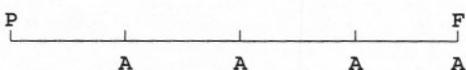
Given: P = \$100
 i = 16% / 4 = 4%
 n = (1)(4) = 4 qtrs.
 F = _____

$$\begin{aligned}
 F &= P(FVM) \\
 &= 100(1.170) \text{ see table} \\
 &= 117
 \end{aligned}$$

Note: \$117 future dollars are worth \$100 in the present, and \$100 of present dollars are \$117 future dollars.

- C. **Annuity** (> 1 payment, > 1 interest calculation) Problem: Calculate the Present Value and Future Value of four \$100 payments, one made every 3 months. Use 16% interest compounded quarterly.

Given A = \$100
 i = 16% / 4 = 4%
 n = (1)(4) = 4 qtrs.
 P = _____



$$\begin{aligned}
 P &= A(PVMA) \\
 &= \$100(3.630) \text{ see table} \\
 &= \$363.00
 \end{aligned}$$

Given A = \$100
 i = 16% / 4 = 4%
 n = (1)(4) = 4 qtrs.
 F = _____

$$\begin{aligned}
 F &= A(FVMA) \\
 &= \$100(4.246) \text{ see table} \\
 &= \$424.60
 \end{aligned}$$

Note: The \$400 in payments are worth less than \$400 if brought back and are worth more than \$400 if brought forward.

PVM Present Value Multiple				
n	4%	6%	10%	12%
1	.9615	.9434	.909	.893
2	.9246	.8899	.826	.797
3	.8889	.8396	.751	.7118
4 B-->	.8548	.7921	.683	.6355
5	.8219	.7473	.621	.5674
6	.7903	.7049	.564	.5066
7	.7599	.6651	.513	.4524
8	.7307	.6274	.467	.4039
20	.4564	.3118	.149	.1037
25	.3751	.2330	.092	.0588
30	.3083	.1741	.057	.0034

<-- 3.

FVM Future Value Multiple				
n	4%	6%	10%	12%
1	1.040	1.060	1.100	1.120
2	1.082	1.124	1.210	1.254
3	1.125	1.191	1.331	1.405
4 B-->	1.170	1.262	1.464	1.574
5	1.217	1.338	1.611	1.762
6	1.265	1.419	1.772	1.974
7	1.316	1.504	1.994	2.211
8	1.369	1.594	2.144	2.476
20	2.191	3.207	6.727	9.646
25	2.666	4.292	10.835	17.000
30	3.243	5.743	17.449	29.960

<-- 4.

PVMA Present Value Multiple Annuity				
n	4%	6%	10%	12%
1	.962	.943	.909	.893
2	1.886	1.833	1.736	1.690
3	2.775	2.673	2.487	2.402
4 C-->	3.630	3.465	3.170	3.037
5	4.452	4.212	3.791	3.605
6	5.242	4.917	4.355	4.111
7	6.002	5.582	4.868	4.564
8	6.733	6.210	5.352	4.968
20	13.590	11.470	8.514	7.469
25	15.622	12.783	9.077	7.843
30	17.292	13.765	9.427	8.055

<-- 6.

<-- 1.

FVMA Future Value Multiple Annuity				
n	4%	6%	10%	12%
1	1.000	1.000	1.000	1.000
2	2.040	2.060	2.100	2.120
3	3.122	3.184	3.310	3.374
4 C-->	4.246	4.375	4.641	4.779
5	5.416	5.637	6.105	6.353
6	6.633	6.975	7.711	8.115
7	7.898	8.394	9.487	10.089
8	9.214	9.897	11.436	12.300
20	29.778	36.785	57.275	72.052
25	41.646	54.865	98.347	133.33
30	56.085	79.058	164.490	241.33

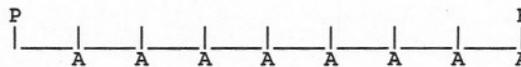
<-- 2.

Question: Assume someone won exactly \$1,000,000 in their state lottery, 20 payments of \$50,000 beginning in one year. Funds invested earned 12% compounded annually. Using the above tables calculate:

Note: These are ordinary annuity tables, which means the equal payments are made at the end of each period. With exact annuity tables, payments would be at the beginning of each period. Most business problems require ordinary tables.

- The value of the annuity today.
- The value of the annuity if all funds received are invested.
- What is the value today of your answer to question 2?
- What is the value in twenty years of your answer to question 3?
- In actuality your answers are all _____.

Note: Answers to 3 and 4 have been adjusted for decimal discrepancies.



Answer:

GIVEN: A = \$50,000 i = 12% compounded annually n = 20 time periods

Note: Annuity is brought back.

$$1. \quad P = A(PVMA)$$

$$P = (\$50,000)(7.469)$$

$$P = \$373,450$$

$$2. \quad F = A(FVMA)$$

$$F = \$50,000(72.052)$$

$$F = \$3,602,600$$

Note: Annuity is brought forward.

Note: Lump sum is brought back.

$$3. \quad P = F(PVM)$$

$$P = \$3,602,600(.1037)$$

$$P = \$373,450$$

$$4. \quad F = P(FVM)$$

$$F = \$373,450(9.646)$$

$$F = \$3,602,600$$

Note: Lump sum is brought forward.

5. Equal

Question: The interest earned on an investment is called the **Internal Rate of Return (IRR)**. Suppose a \$100,000 machine bought today will generate a net return of \$20,128.82 per year for 8 years. In this simplified example, you are to assume all expenses and revenues flow at the end of the year and that taxes and depreciation are ignored. To calculate IRR solve $P = A(PVMA)$ for PVMA. Look your answer up in the PVMA table for 8 years and locate the corresponding interest rate.

Answer:

$$6. \quad P = A(PVMA)$$

$$\$100,000 = \$20,128.82(PVMA)$$

$$PVMA = 4.968 \text{ -----> } 12\% \text{ IRR Compounded Annually}$$